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Alexandre Seuret, Christophe Prieur, Sophie Tarbouriech, Luca Zaccarian

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A. Seuret [*] **C. Prieur** [***] **S. Tarbouriech** [*] **L. Zaccarian** [*,**]

[*] *LAAS-CNRS, Université de Toulouse, CNRS, Toulouse, France (email: aseuret@laas.fr, tarbour@laas.fr, zaccarian@laas.fr).*
[**] *Dipartimento di Ing. Industriale, University of Trento, Italy.*
[***] *Gipsa-lab, Department of Automatic Control, Grenoble Campus, 11 rue des Mathématiques, BP 46, 38402 Saint Martin d'Hères, France (email: christophe.prieur@gipsa-lab.fr).*

Abstract: This paper deals with a systematic way to design the event-triggered rules to stabilize a class of linear reset control systems. The event-triggering condition depends only on local information, that is it only uses the measured signals. The approach proposed combines a hybrid framework to describe the sampled-data system with Lyapunov-based techniques. Dwell-time dependent constructive conditions expressed through linear matrix inequalities (LMI) are proposed to design the event-triggered rule ensuring the asymptotic stability of the closed-loop system. The effectiveness of the approach is evaluated through an example borrowed from the literature.

Keywords: Reset control systems, hybrid systems, event-triggered control, Lyapunov techniques

1. INTRODUCTION

In recent years, sampled-data control designs for linear or nonlinear plants have been studied through several works. Hence, robust stability analysis with respect to aperiodic sampling has been widely studied (see, for example, Chen and Francis (1995); Heemels et al. (2010); Nešić and Teel (2004) and references therein), where variations on the sampling intervals are seen as a disturbance to the periodic case. The objective is then to provide an analysis of such systems using the discrete-time approach (Heemels et al. (2010); Cloosterman et al. (2010)), the input delay approach (Fridman et al. (2004); Seuret (2012)), or the impulsive systems approach (Naghshtabrizi et al. (2008)). Furthermore, an alternative and interesting vision of sampled-data systems has been proposed in Årzén (1999); Åström and Bernhardsson (1999), suggesting to adapt the sampling sequence to certain events related to the state evolution (see, for example, Aström (2008); Heemels et al. (2013); Hespanha et al. (2007); Lunze and Lehmann (2010); Tabuada (2007); Zampieri (2008)). This is called "event-triggered sampling", which naturally mixes continuous and discrete-time dynamics. Thus, the event-triggered algorithm design can be first rewritten as the stability study of a hybrid dynamical system, which has been carried out in different contexts in Goebel et al. (2009, 2012); Prieur et al. (2007, 2010).

In the context of event-triggered control, two objectives can be pursued: 1) the controller is a priori designed and only the event-triggered rules have to be designed, or 2) the joint design of the control law and the event-triggering conditions has to be performed. The first case is called the emulation approach, whereas the second one corresponds to the co-design problem. A large part of the existing works is dedicated to the design of efficient event-triggering rules, that is the designs done by emulation: see, for example, Heemels et al. (2012), Wang and Lemmon (2008), Postoyan et al. (2011), Tallapragada and Chopra (2012), Abdelrahim et al. (2014b) and references therein. Moreover, most of the result on event-triggered control consider that the full state is available, which can be unrealistic from a practical point of view. Hence, it is interesting to address the design of event-triggered controllers by using only measured signals. Some works have addressed this challenge as, for example, in Sbarbaro et al. (2014) in which the dynamic controller is an observer-based one, Abdelrahim et al. (2014a), in which the co-design of the output feedback law and the event-triggering conditions is addressed by using the hybrid framework.

The results proposed in the current paper take place in the context of the emulation approach, when the predesigned controller is issued from a hybrid dynamic output feedback controller, with the aim of using only the available signals. The controller under consideration is a reset control system (see Fichera et al. (2012), Fichera et al. (2013)). Actually, the approach proposed combines a hybrid framework to describe the sampled-data system with Lyapunov-based techniques. Constructive conditions, in the sense that linear matrix inequality (LMI) conditions are associated to a convex optimization scheme, are proposed to design the event-triggered rule ensuring asymptotic stability of the closed-loop system. Furthermore, differently from Abdelrahim et al. (2014a), a condition involving the allowable maximal sampling period T can be deduced by solving

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a set of LMIs proposed using a similar approach as in Mazo et al. (2010). Let us also emphasize that differently from most of the results in the literature, a reset rule is considered in our approach. Through an illustrative example borrowed from the literature, we point out the interest of the reset control law to reduce the number of control updates.

The paper is organized as follows. In Section 2, the system under consideration together with the sampled-data architecture is defined. Describing the associated dynamical hybrid system, the problem we intend to solve is formally stated. Section 3 is dedicated to presenting the main conditions, allowing to design the event-triggering rules. The condition to design the associated dwell-time is also derived. Section 4 illustrates the results and compares them with some existing approach. Finally, in Section 5, some concluding remarks end the paper.

Notation. The sets $\mathbb{N}$, $\mathbb{R}^+$, $\mathbb{R}^n$, $\mathbb{R}^{n\times n}$ and $\mathbb{S}^n$ denote respectively the sets of positive integers, positive scalars, n-dimensional vectors, $n \times n$ matrices and symmetric matrices in $\mathbb{R}^{n\times n}$. For a matrix P in $\mathbb{S}^n$, the notation $P \geq 0$ ($P > 0$) means that P is symmetric positive (definite). The superscript 'T' stands for matrix transposition, and the notation $\mathrm{He}(P)$ stands for $P + P^T$. The Euclidean norm is denoted $|\cdot|$. Given a compact set $\mathcal{A}$, the notation $|x|_{\mathcal{A}} = \min\{|x-y|,\ y \in \mathcal{A}]\}$ indicates the distance of the vector x from the set $\mathcal{A}$. The symbols I and 0 represent the identity and the zero matrices of appropriate dimensions.

2. PROBLEM STATEMENT AND SAMPLED-DATA ARCHITECTURES

2.1 Reset control systems

Consider a continuous-time reset control system described by

$$\begin{cases} \dot{x}_p = A_p x_p + B_p u_p, \\ \dot{x}_c = A_c x_c + B_c y_p, \end{cases} \quad (x_p, x_c) \in \mathcal{C},$$
$$\begin{cases} x_p^+ = x_p, \\ x_c^+ = J_p y_p + J_c x_c, \end{cases} \quad (x_p, x_c) \in \mathcal{D}, \tag{1}$$
$$\begin{cases} u_p = C_c x_c + D_c C_p x_p, \\ y_p = C_p x_p, \end{cases}$$

where $x_p \in \mathbb{R}^{n_p}$, $x_c \in \mathbb{R}^{n_c}$, $u_p \in \mathbb{R}^{m_p}$ and $y_p \in \mathbb{R}^{p_p}$ stand respectively for the state variable of the plant, the state of the dynamic controller, the input and the output of the plant. The matrices $J_p, J_c, A_p, B_p, C_p, A_c, B_c, C_c$ and D_c are constant and given matrices of appropriate dimensions. $\mathcal{C}$ and $\mathcal{D}$ are the flow and jump sets, that are usually defined as

$$\mathcal{C} = \left\{(x_p, x_c) : \begin{bmatrix} y_p \\ x_c \end{bmatrix}^T \bar{M} \begin{bmatrix} y_p \\ x_c \end{bmatrix} \leq 0\right\}, \tag{2a}$$
$$\mathcal{D} = \left\{(x_p, x_c) : \begin{bmatrix} y_p \\ x_c \end{bmatrix}^T \bar{M} \begin{bmatrix} y_p \\ x_c \end{bmatrix} \geq 0\right\}, \tag{2b}$$

where matrix $\bar{M} \in \mathbb{R}^{(p_p+n_c)\times(p_p+n_c)}$ is a design parameter.

Such a system can appear when we connect, for instance, a linear continuous plant with a reset controller (see Fichera et al. (2012), Fichera et al. (2013)). Then, to study this kind of systems, the hybrid formalism of Goebel et al. (2009); Prieur et al. (2007, 2013) can be used.

In this paper we deal with the problem of event-triggered implementation of a stabilizing control law connected to a linear continuous plant. Then, we will show that this kind of problem can be performed by using the framework associated to system (1). Furthermore, to particularize system (1) to the sampled-data architecture, we will see that defining an augmented state composed of the state of the closed-loop system (i.e. x_p and x_c) and the variables due to the sampled part (i.e. the held value of the control input and a timer), the problem consists in designing the sets $\mathcal{C}$ and $\mathcal{D}$.

2.2 Hybrid representation of sampled-data systems

A sampled-data implementation of a control law u_p and of the plant output y_p corresponds to breaking the continuous-time closed loop given by $s(t) = \begin{bmatrix} u_p(t) \\ y_p(t) \end{bmatrix}$, for all $t \geq 0$, and converting this into a zero order hold $\dot{s} = 0$ combined with the update rule $s^+ = \begin{bmatrix} u_p \\ y_p \end{bmatrix}$, which should be performed at suitable times according to the specific sampled-data architecture.

Event-triggered sampling corresponds to performing the update rule s^+ whenever the augmented state (x_p, x_c, s) belongs to suitable sets that should be designed in such a way to guarantee asymptotic stability of the closed-loop sampled-data system. In this case, the sampled-data system may be represented similarly to system (1) by the following dynamics, augmented with a timer σ used to induce a desirable dwell-time between each pair of consecutive samplings:

$$\begin{cases} \dot{x}_p = A_p x_p + B_p s_p, \\ \dot{x}_c = A_c x_c + B_c s_c, \\ \dot{s}_p = 0, \\ \dot{s}_c = 0, \\ \dot{\sigma} = g_T(\sigma), \end{cases} \quad (x_p, x_c, s_p, s_c, \sigma) \in \mathcal{C},$$
$$\begin{cases} x_p^+ = x_p, \\ x_c^+ = J_p C_p x_p + J_c x_c, \\ s_p^+ = C_c x_c + D_c C_p x_p, \\ s_c^+ = C_p x_p, \\ \sigma^+ = 0, \end{cases} \quad (x_p, x_c, s_p, s_c, \sigma) \in \mathcal{D}. \tag{3}$$

Timer $\sigma \in [0, 2T]$ flows by keeping track of the elapsed time since the last sample (where it was reset to zero) according to the following set-valued dynamics:

$$g_T(\sigma) = \begin{cases} 1 & \sigma \leq 2T \\ [0,1] & \sigma = 2T, \end{cases}$$

whose rationale is that whenever $\sigma < 2T$, its value exactly represents the elapsed time since the last sample, moreover $\sigma \in [T, 2T]$ implies that at least T seconds have elapsed since the last sample. [1] In (3), the so-called flow and jump sets $\mathcal{C}$ and $\mathcal{D}$ must be suitably selected to induce a desirable behavior on the sampled-data system and are the available degrees of freedom in the design of the event-triggered algorithm.

[1] Note that the use of a set-valued map for the right hand side g_T of the flow equation for σ enables us to confine the timer σ to a compact set $[0, 2T]$, while at the same time using dynamics whose right hand sides are outer semicountinuous set-valued mappings, thereby satisfying the regularity conditions in (Goebel et al., 2012, As. 6.5) and enjoying the desirable robustness properties of stability of compact attractors established in (Goebel et al., 2012, Ch. 7).

2.3 A compact hybrid dynamical system model

By defining $n = n_p + n_c$, $m = m_p + p_p$ and $p = p_p + n_c$ and the following:

$$\begin{aligned}
&x = \begin{bmatrix} x_p \\ x_c \end{bmatrix} \in \mathbb{R}^n, \; y = \begin{bmatrix} y_p \\ x_c \end{bmatrix} \in \mathbb{R}^p, \; s \in \mathbb{R}^m \\
&A = \begin{bmatrix} A_p & 0 \\ 0 & A_c \end{bmatrix} \in \mathbb{R}^{n\times n}, \qquad B = \begin{bmatrix} B_p & 0 \\ 0 & B_c \end{bmatrix} \in \mathbb{R}^{n\times m}, \\
&C = \begin{bmatrix} C_p & 0 \\ 0 & I \end{bmatrix} \in \mathbb{R}^{p\times n}, \qquad K = \begin{bmatrix} D_c & C_c \\ I & 0 \end{bmatrix} \in \mathbb{R}^{m\times p}, \\
&J = \begin{bmatrix} I & 0 \\ J_p C_p & J_c \end{bmatrix} \in \mathbb{R}^{n\times n},
\end{aligned} \tag{4}$$

system (3) can be rewritten as

$$\begin{cases} \dot{x} &= Ax + Bs, \\ \dot{s} &= 0, \qquad\qquad (x,s,\sigma) \in \mathcal{C}, \\ \dot{\sigma} &= g_T(\sigma), \end{cases}$$
$$\begin{cases} x^+ &= Jx, \\ s^+ &= KCx, \qquad (x,s,\sigma) \in \mathcal{D}, \\ \sigma^+ &= 0, \end{cases} \tag{5}$$

in which we keep separate the variables x and s. Note that one gets $y = Cx$.

In this situation, the definition of the flow and jump sets presented in (2) in the context of reset systems must be adapted, based on the available information at the controller side.

Remark 1. This formulation using hybrid dynamical systems includes the case of periodic sampling which corresponds to performing the update rule $s^+ = \left[\begin{smallmatrix} u_p \\ y_p \end{smallmatrix}\right] = KCx$ at periodic instants of time. Following, e.g., (Goebel et al., 2012, Example 1.4), the corresponding closed loop can be described by modifying system (5) using $\dot{\sigma} = 1$, and selecting the flow and jump sets as

$$\begin{aligned} \mathcal{C} &= \mathbb{R}^n \times \mathbb{R}^m \times [0,T], \\ \mathcal{D} &= \mathbb{R}^n \times \mathbb{R}^m \times \{T\}. \end{aligned}$$

Note that also here timer σ is confined to a compact set $[0,T]$.

Since $\sigma^+ = 0$ across jumps, all solutions have to flow for at least T ordinary time after each jump (a dwell-time T is satisfied by all solutions). This avoids Zeno solutions and also simplifies the implementation. A drawback is that, in general, the origin of resulting system is not asymptotically stable. Some conservative estimates of the range of values of T preserving asymptotic stability can be computed by using the results in Seuret (2012). However, to reduce the average number of samplings per unit of time, one needs to resort to alternative schemes such as the ones described in the following. ⌟

The problem we intend to solve can be summarized as follows.

Problem 1. Given a linear plant and a hybrid controller, that is given matrices A, B, K, C and J. Design event-triggering rules that make the closed-loop system (5) globally asymptotically stable to a compact set wherein $x = 0$.

Note that this problem is an emulation problem (see, for example Heemels et al. (2012), Wang and Lemmon (2008), Postoyan et al. (2011), Tallapragada and Chopra (2012) and the references therein) since we assume that the controller is given. Nevertheless, in the sequel we will discuss some cases where J may take different values (for example as in Fichera et al. (2012)).

3. EVENT-TRIGGERED DESIGN

In order to address Problem 1, we focus on hybrid dynamics (4)-(5) for suitably selecting the flow and jump sets $\mathcal{C}$ and $\mathcal{D}$ whose role is precisely to rule when a sampling should happen, based on the signals available to the controller, namely output y, the last sampled input s and timer σ.

Moreover, in order to ensure that the proposed solution does not sample too often, we will prevent sampling unless $\sigma = T$ (namely unless at least T ordinary time has elapsed since the last sample). Then we select the following sets $\mathcal{C}$ and $\mathcal{D}$:

$$\mathcal{C} = \mathcal{F} \cup \{\sigma \in [0,T]\} \tag{6a}$$
$$\mathcal{D} = \mathcal{J} \cap \{\sigma \in [T,2T]\}, \tag{6b}$$

where sets $\mathcal{F}$ and $\mathcal{J}$ are selected as follows:

$$\mathcal{F} = \left\{ (x,s) : \begin{bmatrix} y \\ s - Ky \end{bmatrix}^T M \begin{bmatrix} y \\ s - Ky \end{bmatrix} \le 0 \right\}, \tag{6c}$$
$$\mathcal{J} = \left\{ (x,s) : \begin{bmatrix} y \\ s - Ky \end{bmatrix}^T M \begin{bmatrix} y \\ s - Ky \end{bmatrix} \ge 0 \right\}, \tag{6d}$$

with matrix $M = \left[\begin{smallmatrix} M_1 & M_2 \\ M_2^T & M_3 \end{smallmatrix}\right] \in \mathbb{R}^{(p+m)\times(p+m)}$ to be designed. Solution (6) to the presented event-triggered problem is parametrized by M and T.

Note that the jump set selection in (6b) ensures that all solutions satisfy a dwell-time constraint corresponding to T. Indeed, jumps are inhibited unless timer $\sigma \ge T$, which implies that at least T ordinary time elapses between each pair of consecutive sampling times. In the following developments, the dwell-time T will be a design parameter for the design of the matrix M that defines the flow and jump sets.

Remark 2. The definition of the flow and jump sets provided in (6) meets the one provided in the recent paper Postoyan and Girard (2015). The novelty of this definition relies on the consideration of a general matrix M. For example, selecting $M_2 = 0$ leads to definition of the flow and jump sets usually employed in the literature issued from an Input-to-State (or Input-to-Output) analysis. See Postoyan and Girard (2015) for more details. ⌟

The effectiveness of the proposed solution is established and proven in our main result, reported next, which is based on the existence of a strict hybrid Lyapunov function, namely a Lyapunov function that decreases both during flow and across jumps (samplings) of the proposed event-triggered sampled-data system.

Theorem 1. Assume that there exist matrices $P = P^T > 0 \in \mathbb{R}^{n\times n}$, $M = \left[\begin{smallmatrix} M_1 & M_2 \\ M_2^T & M_3 \end{smallmatrix}\right] \in \mathbb{R}^{(p+m)\times(p+m)}$, satisfying constraints

$$\Psi_M < 0, \qquad \Phi_M < 0, \qquad \Theta_M < 0, \tag{7a}$$

where

$$
\begin{aligned}
\Psi_M &= \begin{bmatrix} \mathrm{He}(PA_{cl}) - C^T M_1 C & PB - C^T M_2 \\ B^T P - M_2^T C & -M_3 \end{bmatrix}, \\
\Phi_M &= \begin{bmatrix} I \\ 0 \end{bmatrix}^T M \begin{bmatrix} I \\ 0 \end{bmatrix} = M_1, \\
\Theta_M &= \left(\Lambda(T) \begin{bmatrix} J \\ KC \end{bmatrix}\right)^T P \left(\Lambda(T) \begin{bmatrix} J \\ KC \end{bmatrix}\right) - P,
\end{aligned} \tag{7b}
$$

and

$$
\begin{aligned}
A_{cl} &= A + BKC \\
\Lambda(T) &= [\, I \ 0 \,]\, e^{A_f T},
\end{aligned} \tag{7c}
$$

with $A_f = \left[\begin{smallmatrix} A & B \\ 0 & 0 \end{smallmatrix}\right] \in \mathbb{R}^{(n+m)\times(n+m)}$.

Then there exists a positive scalar $\rho > 0$ such that the compact attractor:

$$
\mathcal{A} := \{(x, s, \sigma) : \ x = 0, s = 0, \sigma \in [0, 2T]\}, \tag{8}
$$

is uniformly globally asymptotically stable (UGAS) for the closed-loop dynamics (5), (6). [2]

Remark 3. As mentioned earlier, the dwell-time T appears as a tuning parameter of the event-triggered control system (5)-(6). Contrary to most of the approaches developed in the literature, where the dwell-time is computed a posteriori, Theorem 1 includes the guaranteed dwell-time T. Therefore, if one can find a solution to the three LMI conditions for a given parameter T, then this same T can be employed in the definition of the flow and jumps sets (6). ⌟

Remark 4. (Optimized design) From the LMI $\Psi_M < 0$ in (7a), the matrices $\mathrm{He}(PA_{cl}) - C^T M_1 C$ and $-M_3$ are required to be negative definite. Then a natural optimization procedure consists in the minimization of the effect of the non-diagonal term $PB - C^T M_2$. This optimization can be performed by minimizing the size of matrix M_3. This problem can be reformulated in terms of an LMI optimization as follows

$$
\begin{aligned}
&\min_{P,M} Tr(M_3), \text{ subject to:} \\
&\quad P > I, \ \Psi_M < 0, \ \Phi_M < 0, \ \Theta_M < 0.
\end{aligned} \tag{9}
$$

In the previous problem, the additional constraint $P > I$ has been imposed for well conditioning the LMI constraints. Another interpretation of this optimization problem is that minimizing the trace of M_3 aims at increasing the negativity of matrix M_3, which, in turns, leads to larger flow sets (see equation (6)), because the set of vectors x for which $x^T M_3 x < 0$ becomes increasingly larger for matrices M_3 with decreasing trace. Therefore it is expected that solutions will flow longer and jump less in light of larger flow sets. ⌟

4. EXAMPLE

Let us specify that, in this section, we have added to the simulation a scalar timer variable τ exactly measuring the elapsed time since the last jump. This means that $\dot{\tau} = 1$ while flowing and $\tau^+ = 0$ after jumping. Then the maximum value of τ illustrates the length of the sampling intervals.

Consider a linear plant coupled with a dynamic output feedback controller borrowed from Donkers and Heemels (2012) and Abdelrahim et al. (2014a). The plant is defined by

$$
\begin{cases}
\dot{x}_p = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_p, \\
y_p = \begin{bmatrix} -1 & 4 \end{bmatrix} x_p.
\end{cases} \tag{10}
$$

and the dynamic output feedback controller is given by

$$
\begin{cases}
\dot{x}_c = \begin{bmatrix} 1.0919 & -1.1422 \\ 4.9734 & -6.1425 \end{bmatrix} x_c + \begin{bmatrix} 16.7501 \\ 64.6472 \end{bmatrix} y_p, \\
u_p = \begin{bmatrix} 0.01157 & -0.0928 \end{bmatrix} x_c.
\end{cases} \tag{11}
$$

The dynamic output feedback controller characterized through the matrices (A_c, B_c, C_c, D_c) above has been obtained using an optimization process provided in Abdelrahim et al. (2014a). Note that these matrices only correspond to the dynamics of the controller while flowing. In order to complete the description of the controller, we recall the following equations representing the evolution of the system state while jumping (according to (5))

$$
\begin{cases}
x_p^+ = x_p, \\
x_c^+ = J_c x_c + J_p C_p x_p.
\end{cases}
$$

The controller employed in Abdelrahim et al. (2014a) implicitly uses the following matrices $J_c = I$ and $J_p = 0$, which resume the second equation to

$$
x_c^+ = x_c.
$$

This controller has already shown some improvements with respect to the literature (for instance with respect to Donkers and Heemels (2012)). Indeed, the authors obtained a dwell-time $T = 0.0114s$. Note that solutions to the conditions of Theorem 1 exist for values of the design parameter T up to $0.11s$, which is ten times larger than the solution provided in Abdelrahim et al. (2014a).

In order to show the efficiency of the reset control system approach for event-triggered control, we introduce an additional contribution to the controller consisting of the same matrix $J_c = I$ associated with $J_p = 0.01\,[0.2 \ \ -1]^T$, leading to the following equation

$$
x_c^+ = x_c + J_p C_p x_p.
$$

Again, solutions to the LMI problem (7) exist for values of T up to $0.11s$. Note that the addition of the matrix J_p introduces constraints in (7) that make the conditions infeasible for values of T smaller than $0.02s$.

Figure 1 depicts several simulations obtained with the initial conditions $x_{p0} = [10 \ \ -5]^T$ and $x_{c0} = [0 \ 0]^T$. This figure shows the timer τ corresponding to these simulations where the matrix M has been obtained for several values of $T = 0.02, 0.04$ and $0.08s$ for the two controllers ($J_p = 0$ and $J_p \neq 0$) in order to show the variation of the response for different parameter selections. At the bottom of each figure, the notation N denotes the number of control updates that occurs during the simulation time of $20s$. As a first comment on this figure, one can see that increasing T (from left to right in the figure) leads to a notable reduction of the number of control updates. Second, Figure 1 shows that the addition of matrix J_p to the controller reduces the number of control updates for all values of T and for this initial condition, proving the potential of the introduction of J_p.

It should also be noticed that the reduction of the number of control updates has an impact on the performance of

[2] The proof was removed from reason of place but can be obtained from the authors.

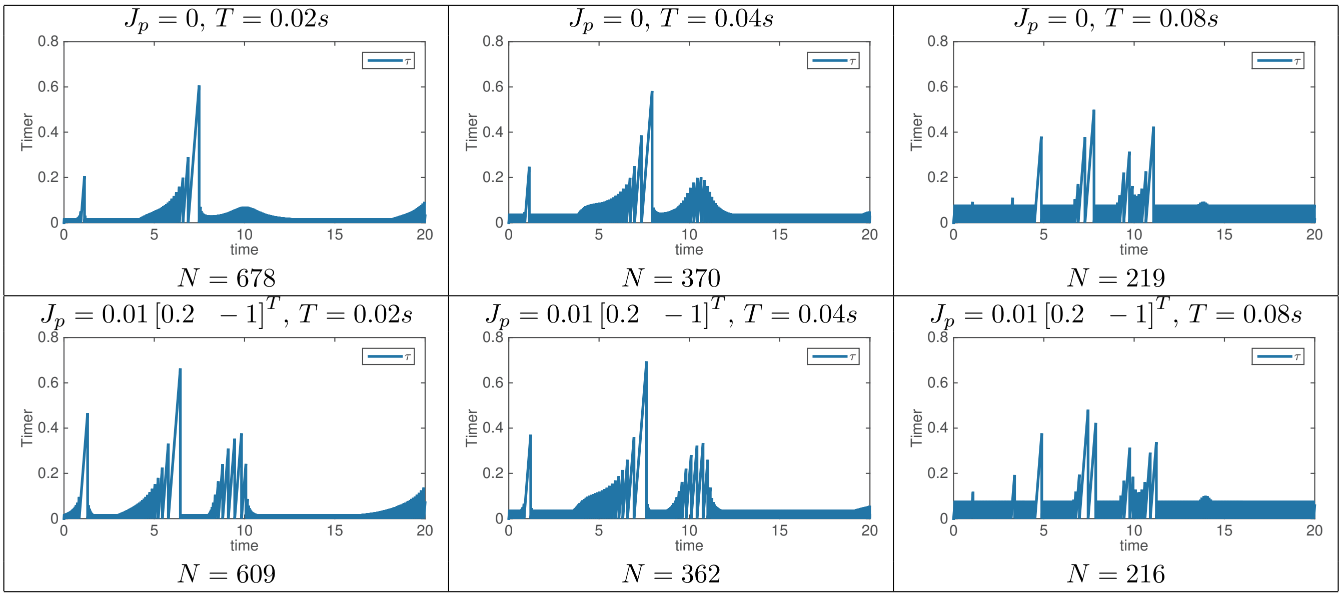

Fig. 1. Figure representing the timer τ and the control input s issued from Theorem 1 with $T = 0.02$ (left), $T = 0.04$ (middle) and $T = 0.08$ (right) and for $J_p = 0$ (top) and $J_p = 0.01\,[0.2 \quad -1]^T$ (bottom).

the controller. In order to illustrate this fact, Figure 2 depicts the state, the timer and the control input of the closed-loop system with $J_p = 0.01\,[0.2 \quad -1]^T$. While the first simulation with $T = 0.02s$ (top three plots) leads to 609 control updates and the second one with $T = 0.1s$ (bottom three plots) leads to 187 control updates, the state depicted in the first simulation with $T = 0.02s$ (top) is smoother than the second one with $T = 0.1s$ (bottom), which shows an oscillatory behavior. This can be interpreted in terms of the classical trade-off between the number of control updates and the performance of the closed-loop system.

5. CONCLUSION

In this paper, we provided a stability theorem for linear systems controlled using a dynamic output feedback reset controller. The contribution is twofold. On the first hand, we have extended the class of controllers usually employed in the literature of event-triggered control by introducing an additional contribution in the jump dynamics (which is due to the reset part of the controller). Second, the stability conditions depends on a dwell-time T, which appears as an explicit tuning parameter for the selection of the flow and jump sets.

Future work involves providing more advanced theoretical conditions in order to address the co-design (namely simultaneously designing the feedback stabilizer and its event-triggered sampled data implementation).

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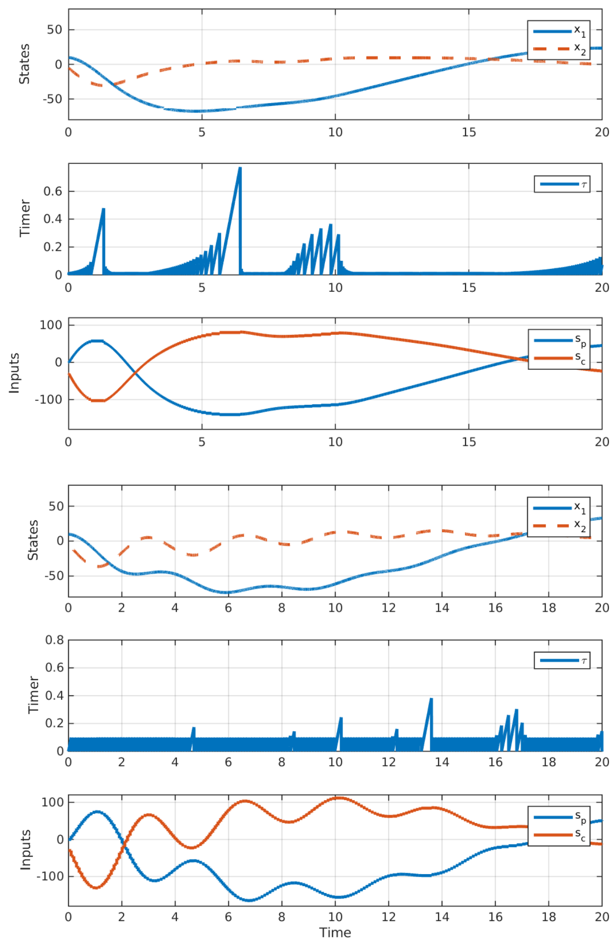

Fig. 2. Plant state variables x_p, timer τ and inputs $s = (s_p, s_c)$ of the closed-loop (5), (6), using Theorem 1 with $T = 0.02s$ (top) and $T = 0.1s$ (bottom).

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